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Diffraction of blast wave for the oblique case

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In this paper the problem of interaction of an oblique blast wave encountering a small bend along a plane wall is studied for the case when relative outflow from the reflected shock is supersonic. A theoretical solution is developed completely with the help of conformal transformations and complex variable techniques. Numerical results showing the pressure distribution along the wall have been obtained for two incident shock strengths.

Introduction

Lighthill (1949) considered the diffraction of a plane shock travelling parallel to a wall and meeting a corner where the wall turns through a small angle. The analogous problem for a plane shock hitting the wall obliquely, together with the associated reflected shock has been considered by Srivastava (1968) by employing the techniques developed by Lighthill. Srivastava & Ballabh (1955) established that the region within the incident and reflected shock remains undisturbed after the shock configuration has crossed the bend. Srivastava (1968) developed the mathematical theory for the cases when the relative outflow from the reflected shock is subsonic and sonic. The present investigation is concerned with the diffraction effects when the relative outflow behind the reflected shock before diffraction is supersonic. In fact this work is the continuation of Srivastava's work, and the method of solution is on the same lines, with necessary modifications. The main feature of the present problem is that the non-uniform region is enclosed by the arcs of the Mach circle, the wall and the reflected-diffracted shock. The region of disturbance is not symmetric about the corner.

Lighthill (1950) encounters a similar situation where the non-uniform region, having symmetry about the corner, is bounded by the arcs of the Mach circle, the wall and the diffracted shock. The disturbed region is transformed into a rectangle and the solution is constructed therein. In the process the shock boundary condition is expanded into a Fourier Series, the resulting solution involving certain infinite products.

In the present case we use an alternative approach and construct the solution in a semi-infinite plane retaining the shock boundary condition in its original form. Numerical curves showing the pressure distribution along the wall for two incident shock strengths have been obtained.

Our attention has been drawn by Lighthill to Ter-Minassiants' (1969) treatment of the same problem. Ter-Minassiants obtains the solution by an ingenious adaptation of Lighthill's (1950) method. As the present investigation is based on an independent approach, we have availed ourselves of the opportunity of comparing the respective results.

Mathematical formulation

The flow variables across the incident and reflected shock before the oblique shock configuration has encountered the bend are related, in the notation of figure 1, as

$$\left.\begin{aligned} &\bar{q}_1 = \tfrac{5}{6}\bar{U}\left(1-\frac{a_0^2}{\bar{U}^2}\right), \quad p_1 = \tfrac{5}{6}\rho_0\left(\bar{U}^2-\frac{a_0^2}{7}\right), \quad \rho_1 = \frac{6\rho_0}{1+(5a_0^2/\bar{U}^2)}, \\ &\bar{q}_2 = \tfrac{5}{6}(U^*-\bar{q}_1)\left\{1-\frac{a_1^2}{(U^*-\bar{q}_1)^2}\right\}+\bar{q}_1, \quad p_2 = \tfrac{5}{6}\rho_1\left\{(U^*-\bar{q}_1)^2-\frac{a_1^2}{7}\right\}, \\ &\rho_2 = \frac{6\rho_1}{1+5a_1^2/(U^*-\bar{q}_1)^2}, \end{aligned}\right\} \tag{1}$$

where $\bar{U} = U\sin\alpha_0, \quad U^* = U\sin\alpha_2, \quad \bar{q}_1 = -q_1\cos(\alpha_0+\alpha_2),$

$$\bar{q}_2 = q_2\sin\alpha_2 \quad \text{and} \quad a = (\gamma p/\rho)^{\frac{1}{2}}.$$

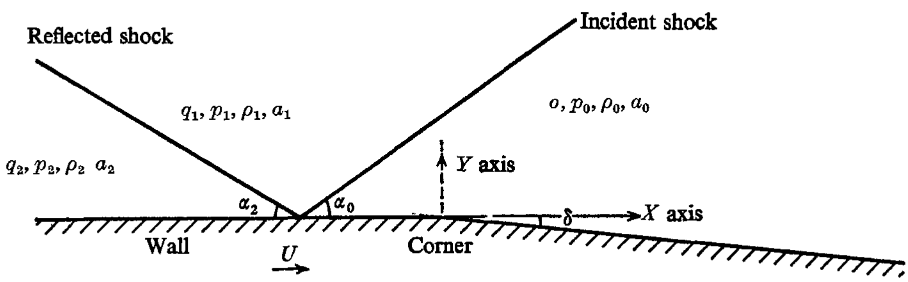

FIGURE 1

Let the pressure, density, velocity and entropy of the flow field behind the reflected-diffracted shock after diffraction be p_2', ρ_2', $\mathbf{q}_2'$ and s_2'. Using small perturbation theory and conical field transformations, the flow equations can be linearized, and they yield a single second-order partial differential equation in p, viz.

$$\left(x\frac{\partial}{\partial x}+y\frac{\partial}{\partial y}+1\right)\left(x\frac{\partial p}{\partial x}+y\frac{\partial p}{\partial y}\right) = \frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}, \tag{2}$$

where $p = \dfrac{p_2'-p_2}{a_2\rho_2 q_2}, \quad x = \dfrac{X-q_2 t}{a_2 t}, \quad y = \dfrac{Y}{a_2 t}$ and $\mathbf{q}_2' = q_2(1+u, v)$.

The characteristics of (2) are tangents to the unit circle $x^2+y^2 = 1$, and therefore the region of disturbance will be bounded by the unit circle, reflected-diffracted shock and the wall. The intersection or otherwise of the unit circle with the reflected-diffracted shock will depend in general on whether $\{(U-q_2)/a_2\}\sin\alpha_2 \lesseqgtr 1$. But it can easily be proved with the help of regular reflexion theory developed by Bleakney & Taub (1949) that we would always have

$$\left(\frac{U-q_2}{a_2}\right)\sin\alpha_2 < 1 \quad \text{as} \quad \frac{U-q_2}{a_2} = \left\{\frac{2(1+\eta'^2x'^2)}{(\gamma+1)\eta'-(\gamma-1)}\right\}^{\frac{1}{2}},$$

$$\eta' = \frac{\rho_2}{\rho_1} > 1 \quad \text{and} \quad x' = \frac{\cot\alpha_2}{\eta'}.$$

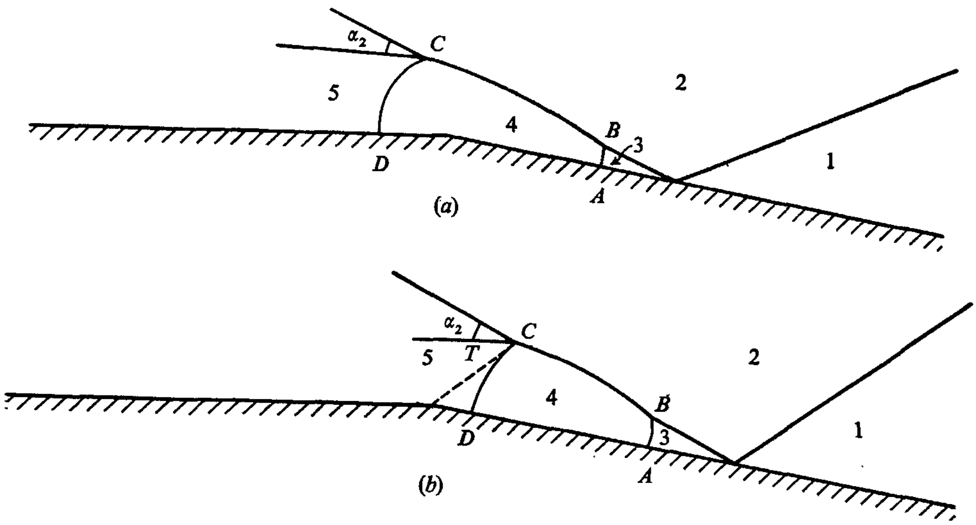

FIGURE 2. (*a*) $(U-q_2)/a_2 > 1$, $q_2/a_2 < 1$; (*b*) $(U-q_2)/a_2 > 1$, $q_2/a_2 > 1$.

The possible configurations are shown in figures 2 (*a*) and (*b*). The flow in region 5 is given by (1), and in region 3, too, (1) holds, but with changed angle of incidence. To determine the flow in the disturbed region, on the other hand, we have to solve (2) under the following boundary conditions:

Following Srivastava (1968), on the shock boundary $x = \kappa - y\cot\alpha_2$ we have

$$\frac{\partial p/\partial x}{\partial p/\partial y} = \frac{y\left\{(\kappa - y\cot\alpha_2) + \dfrac{B-AG(y)}{B_2-A_2G(y)}\right\} - (\kappa - y\cot\alpha_2)\dfrac{B_1-A_1G(y)}{B_2-A_2G(y)}}{\{1-(\kappa-y\cot\alpha_2)^2\}}, \tag{3}$$

where A's and B's are constants and $\kappa = \{(U-q_2)/a_2\}$.

Now, since $v = -\delta$ at A and $v = 0$ at C

$$\int \frac{\partial v}{\partial y} dy = \int_\Gamma \frac{B_1 - A_1G(y)}{B_2 - A_2G(y)} dp = \delta, \tag{4}$$

where Γ denotes the diffracted part of the shock and δ is the bend.

The boundary conditions on the wall and parts of the unit circle depend on whether the corner is within or outside the disturbed region:

(i) Corner included in the region of disturbance ($q_2/a_2 < 1$, figure 2 (*a*)). On the wall $\partial p/\partial y = 0$ except at the corner where

$$\lim_{y\to 0} \int_{-M_2-\epsilon}^{-M_2+\epsilon} \frac{\partial p}{\partial y} dx = M_2\delta.$$

On the arc CD, $p = 0$ and on AB, $p =$ constant.

(ii) Corner excluded from the region of disturbance ($q_2/a_2 > 1$, figure 2(*b*)). On the wall $\partial p/\partial y = 0$, on the arc AB, $p =$ constant, on the arc CT, $p = 0$ and on the arc TD, $p = -M_2\delta/(M_2^2-1)^{\frac{1}{2}}$.

It may, however, be pointed out that in the course of numerical calculations it was observed that the configuration (2(*b*)) arises only for very high shock strengths and that too when the angle of incidence is near the sonic angle (Bleakney & Taub 1949).

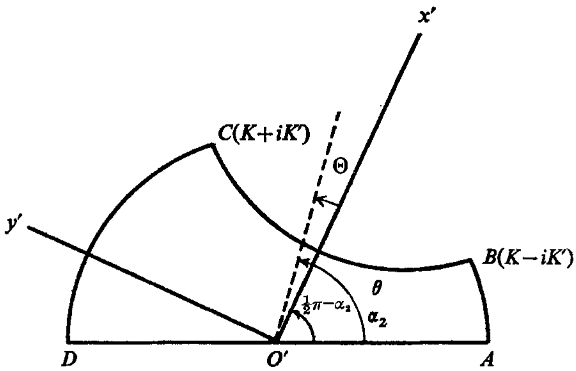

FIGURE 3. ζ-plane.

Solution of the problem

Using Busemann's (1943) transformation,

$$x = r\cos\theta, \quad y = r\sin\theta, \quad \rho = \frac{1-(1-r^2)^{\frac{1}{2}}}{r},$$

(2) becomes Laplace's equation in plane polar co-ordinates, and the boundaries are transformed as shown in figure 3. Each of the angles of the curvilinear quadrilateral bounding the disturbed region is now a right angle. The shock boundary condition becomes (Srivastava 1968, equations (21), (22))

$$\frac{C_1+D_1\tan\Theta+E_1\tan^2\Theta+F_1\tan^3\Theta}{C_1'+D_1'\tan\Theta+E_1'\tan^2\Theta+F_1'\tan^3\Theta} = \frac{K^2}{K'^2}\tan\Theta + \frac{(1-K^2\sec^2\Theta)^{\frac{1}{2}}}{K'^2}\frac{\partial p/\partial n}{\partial p/\partial s}, \quad (5)$$

where $K = \kappa\sin\alpha_2$, $K^2+K'^2 = 1$ and C's, D's, etc., are known constants, $\Theta = \theta-(\frac{1}{2}\pi-\alpha_2)$.

The corner $(-M_2, 0)$ transforms to

$$\xi = -\frac{\{1-(1-M_2^2)^{\frac{1}{2}}\}}{M_2} \quad (\eta = 0)$$

and the condition holding there becomes

$$\lim_{\eta\to 0}\int\frac{\partial p}{\partial\eta}d\xi = \frac{M_2\delta}{(1-M_2^2)^{\frac{1}{2}}}. \quad (6)$$

Now p is given as a harmonic function satisfying certain boundary conditions in the quadrilateral $ABCD$ with AB, BC and CD as circular arcs and DA as straight segment. The solution of this potential problem with the prescribed boundary conditions is obtained by transforming the region bounded by $ABCD$ to semi-infinite medium in the Z_1-plane by the following successive conformal transformations.

(i) The transformation

$$Z_1 = (K+iK')\left\{i - \frac{2K'}{\zeta-(K+iK')}\right\}, \quad \zeta = \rho\, e^{i\Theta}$$

transforms the region bounded by $ABCD$ to quarter space having a semi-circular cut with centre $(0, a+l)$ and radius a (figure 4 (a)), where

$$a = \frac{K'}{K'\sin\alpha_2 + K\cos\alpha_2} \quad \text{and} \quad l = \frac{K-\sin\alpha_2}{1-K\sin\alpha_2+K'\cos\alpha_2}.$$

The circular arc D_1A_1 is the mapping of the wall, A_1B_1 of unit circle, B_1C_1 of the shock front, and C_1D_1 of the other part of the unit circle.

(ii) This quarter space is transformed to the region between two semi-circles (figure 4 (b)) with the help of conformal transformation

$$Z_2 = \frac{a}{(a+l)+iZ_1}.$$

The radius of the inner circle is $b = a/\{2(a+l)\}$ and of the outer one is unity.

(iii) Transformation $Z_3 = (Z_2-\lambda)/(\lambda Z_2 - 1)$ converts the region between two non-concentric semi-circles to the region between two concentric semi-circles of radii unity and $\lambda = [\cos\alpha_2 + (\cos^2\alpha_2 - K'^2)^{\frac{1}{2}}]/K'$ (figure 4 (c)).

(iv) $Z_4 = -\frac{1}{2}\pi + i\log Z_3$ maps the region between two concentric semi-circles to the region bounded by the rectangle (figure 4 (d)) having the corners $A_4(\frac{1}{2}\pi, 0)$ $B_4(\frac{1}{2}\pi, \log\lambda)$, $C_4(-\frac{1}{2}\pi, \log\lambda)$ and $D_4(-\frac{1}{2}\pi, 0)$.

(v) Elliptic function transformation

$$Z_1 = \operatorname{sn}\left(\frac{Z_4}{\alpha'}, \bar\kappa\right) \quad \text{or} \quad \frac{Z_4}{\alpha'} = \int_0^{z_1} \frac{dz_1}{\{(1-z_1^2)(1-\bar\kappa^2 z_1^2)\}^{\frac{1}{2}}}, \tag{7}$$

where $\alpha' = \frac{1}{2}\pi/\bar K(\bar\kappa)$, $\bar\kappa$ is the modulus of transformation satisfying

$$\frac{\bar K'(\bar\kappa)}{\bar K(\bar\kappa)} = \frac{\log\lambda}{\frac{1}{2}\pi} \quad \text{with} \quad \bar K(\bar\kappa) = \int_0^1 \frac{dx_1}{\{(1-x_1^2)(1-\bar\kappa^2 x_1^2)\}^{\frac{1}{2}}} \quad \text{and} \quad \bar K'(\bar\kappa) = \bar K[\surd(1-\bar\kappa^2)]$$

converts the region bounded by the rectangle to a semi-infinite plane (figure 4 (e)). In this transformed z_1-plane,

$-1/\bar\kappa$ to -1 corresponds to the part of the unit circle C_4D_4,

-1 to $+1$ corresponds to the wall D_4A_4,

$+1$ to $+1/\bar\kappa$ corresponds to the other part of the unit circle A_4B_4,

$+1/\bar\kappa$ to $+\infty$ corresponds to the part B_4O of the shock front, while

$-\infty$ to $-1/\bar\kappa$ corresponds to the remaining part OC_4 of the shock

Under these transformations the shock boundary condition (5) reduces to

$$\frac{\partial p/\partial y_1}{\partial p/\partial x_1} = \frac{K'^2C_1 + (K'^2D_1 - K^2C_1')\tan\Theta + (K'^2E_1 - K^2D_1')\tan^2\Theta + (K'^2F_1 - K^2E_1')\tan^3\Theta}{[K'^2 - K^2\tan^2\Theta]^{\frac{1}{2}}[C_1' + D_1'\tan\Theta + E_1'\tan^2\Theta]}$$

$$\text{for} \quad |x_1| > \frac{1}{\bar\kappa} \quad (y_1 = 0). \tag{8}$$

Here $\tan\Theta = (K'/K)(Z_1^2-1)/(Z_1^2+1)$ where Z_1 is replaced in terms of z_1 with the help of transformations (i) to (v).

The corner is transformed successively to

$$\frac{1-(1-M_2^2)^{\frac{1}{2}}}{M_2}\{-\sin\alpha_2+i\cos\alpha_2\} \quad \text{in } \zeta\text{-plane},$$

$$\frac{K'(1-M_2^2)^{\frac{1}{2}}+i(K+M_2\sin\alpha_2)}{1+M_2(K\sin\alpha_2-K'\cos\alpha_2)} \quad \text{in } Z_1\text{-plane},$$

and

$$-\tfrac{1}{2}\pi+\tan^{-1}\left\{\frac{(1-M_2^2)^{\frac{1}{2}}\sqrt{(\cos^2\alpha_2-K'^2)}}{M_2K+\sin\alpha_2}\right\} \quad \text{in } Z_4\text{-plane.}$$

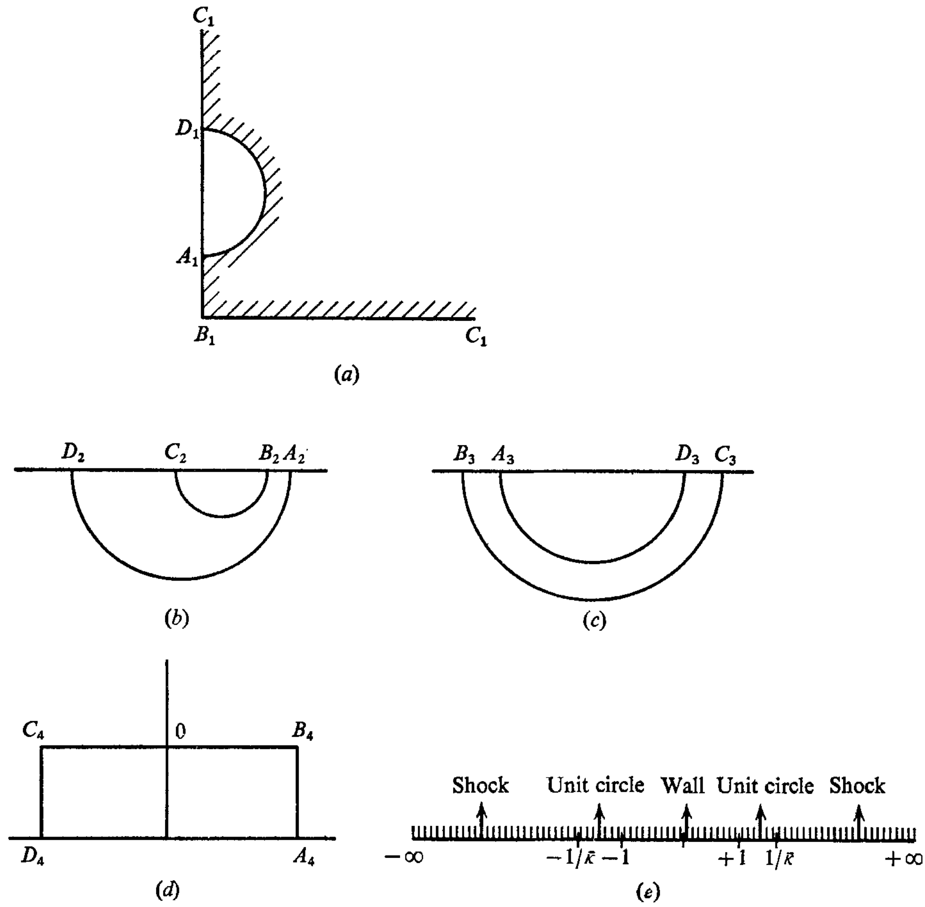

FIGURE 4. (*a*) Z_1-plane, (*b*) Z_2-plane, (*c*) Z_3-plane, (*d*) Z_4-plane, (*e*) z_1-plane.

The co-ordinates of the corner x_0 in the z_1-plane are obtained with the help of transformation (v).

The wall boundary condition becomes $\partial p/\partial y_1 = 0$ for $-1 < x_1 < +1$, $y_1 = 0$, and the discontinuity condition (6) at the corner now becomes

$$\lim_{y_1\to 0}\int \frac{\partial p}{\partial y_1}dx_1 = \frac{M_2\delta}{(1-M_2^2)^{\frac{1}{2}}}. \tag{9}$$

The condition on the parts of the unit circle can be written as $\partial p/\partial x_1 = 0$. But, when $M_2 > 1$, this must be supplemented with the condition

$$\lim_{y_1 \to 0} \int \frac{\partial p}{\partial y_1} dx_1 = -\frac{M_2 \delta}{(M_2^2 - 1)^{\frac{1}{2}}}, \tag{10}$$

which holds at x_0 of the z_1-plane corresponding to the point

$$\frac{1 - i(M_2^2 - 1)^{\frac{1}{2}}}{M_2} \{-\sin\alpha_2 + i\cos\alpha_2\} \quad \text{of the } \zeta\text{-plane.}$$

The point corresponding to x_0 in the Z_4-plane in the case $M_2 > 1$ is

$$Z_4 = -\tfrac{1}{2}\pi + i\log\left\{\frac{(M_2 K + \sin\alpha_2) + (M_2^2 - 1)^{\frac{1}{2}}(\cos^2\alpha_2 - K'^2)^{\frac{1}{2}}}{K + M_2 \sin\alpha_2}\right\}.$$

The solution is obtained by the introduction of the function

$$\omega(z_1) = (\partial p/\partial x_1) - i(\partial p/\partial y_1)$$

which is regular throughout the upper half-plane, since p is harmonic. In terms of $\omega(z_1)$, the discontinuity conditions (9) and (10) can be expressed by saying that, near $z_1 = x_0$,

$$\left.\begin{aligned} \omega(z_1) &\sim \frac{M_2 \delta/\pi(1 - M_2^2)^{\frac{1}{2}}}{z_1 - x_0} \quad (M_2 < 1), \\ \omega(z_1) &\sim \frac{-iM_2 \delta/\pi(M_2^2 - 1)^{\frac{1}{2}}}{z_1 - x_0} \quad (M_2 > 1). \end{aligned}\right\} \tag{11}$$

Equation (4) becomes

$$\int_{-\infty}^{-1/\kappa} \frac{B_1 - A_1 G(y)}{B_2 - A_2 G(y)} \frac{\partial p}{\partial x_1} dx_1 + \int_{1/\kappa}^{+\infty} \frac{B_1 - A_1 G(y)}{B_2 - A_2 G(y)} \frac{\partial p}{\partial x_1} dx_1 = \delta, \tag{12}$$

where $y = K(\cos\alpha_2 + \sin\alpha_2 \tan\Theta)$ and $\tan\Theta$ is replaced in terms of z_1 with the help of conformal transformations.

Now $\log\omega(z_1)$ is a function with known argument on the real axis of the z_1-plane. Therefore, $\omega(z_1)$ is given by Poisson's integral formula,

$$\log\frac{\omega(z_1)}{G_1} = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\tan^{-1}\left\{-\frac{\partial p/\partial y_1}{\partial p/\partial x_1}\right\}_{x_1 = t}}{(t - z_1)} dt, \tag{13}$$

where G_1 is a real constant.

The above integral, when evaluated for points lying on the real axis, will involve two other constants G_2 and H, which creep in due to the discontinuity at $z_1 = x_0$. Finally, the constants G_2 and G_1 merge, and we are left with only two constants G and H, which are determined from the conditions (11) and (12) respectively.

The function $\omega(z_1)$ is completely imaginary on the segments of real axis from $-1/\kappa$ to -1 and $+1$ to $+1/\kappa$, real on -1 to $+1$ and the value of the argument is prescribed on the remaining range by (8). To determine $\omega(z_1)$, the integral on the left-hand side of (13) is broken into five integrals ranging from $-\infty$ to $-1/\kappa$,

$-1/\kappa$ to -1, -1 to $+1$, $+1$ to $+1/\kappa$ and $+1/\kappa$ to $+\infty$. Taking into account the boundary conditions and simplifying, $\omega(z_1)$ is given by

$$\log\frac{\omega(z_1)}{G_1} = \log\frac{G_2\delta[H(z_1-x_0)-1]}{z_1-x_0}\left(\frac{\{z_1^2-(1/\kappa^2)\}}{z_1^2-1}\right)^{\frac{1}{2}}$$

$$+\frac{1}{\pi}\int_{-\infty}^{-1/\kappa}\frac{\tan^{-1}\left\{-\dfrac{\partial p/\partial y_1}{\partial p/\partial x_1}\right\}_{x_1=t}}{t-z_1}dt+\frac{1}{\pi}\int_{+1/\kappa}^{+\infty}\frac{\tan^{-1}\left\{-\dfrac{\partial p/\partial y_1}{\partial p/\partial x_1}\right\}_{x_1=t}}{t-z_1}dt+\frac{1}{\kappa}.$$

Making use of the substitutions

$$t=-\left(\frac{1}{\tau}+\frac{1}{\kappa}-1\right)\quad\text{and}\quad t=\left(\frac{1}{\tau'}+\frac{1}{\kappa}-1\right)$$

in the first and second integrals of the left-hand side, respectively, and merging constant G_1 with G_2, we get

$$\omega(z_1)=\frac{G\delta[H(z_1-x_0)-1]}{z_1-x_0}\left(\frac{(z_1^2-1/\kappa^2)}{z_1^2-1}\right)^{\frac{1}{2}}\left(-z_1-\frac{1}{\kappa}\right)^{\beta/\pi}\left(z_1-\frac{1}{\kappa}\right)^{-\beta'/\pi}e^{\phi}, \quad (14)$$

where
$$\phi=i(\beta+\beta')+\frac{z_1+((1/\kappa)-1)}{12\pi}\left[\frac{f(\tau)_{\tau=0}-\beta}{1}\right.$$
$$+\frac{4\{f(\tau)_{\tau=0\cdot25}-\beta\}}{1+0\cdot25\{z_1+[(1/\kappa)-1]\}}+\frac{2\{f(\tau)_{\tau=0\cdot50}-\beta\}}{1+0\cdot50\{z_1+[(1/\kappa)-1]\}}$$
$$\left.+\frac{4\{f(\tau)_{\tau=0\cdot75}-\beta\}}{1+0\cdot75\{z_1+[(1/\kappa)-1]\}}+\frac{f(\tau)_{\tau=1}-\beta}{1+\{z_1+[(1/\kappa)-1]\}}\right]$$
$$+\frac{z_1-\{(1/\kappa)-1\}}{12\pi}\left[\frac{f(\tau')_{\tau'=0}-\beta'}{1}+\frac{4\{f(\tau')_{\tau=0\cdot25}-\beta'\}}{1-0\cdot25\{z_1-[(1/\kappa)-1]\}}\right.$$
$$+\frac{2\{f(\tau')_{\tau=0\cdot50}-\beta'\}}{1-0\cdot50\{z_1-[(1/\kappa)-1]\}}+\frac{4\{f(\tau')_{\tau'=0\cdot75}-\beta'\}}{1-0\cdot75\{z_1-[(1/\kappa)-1]\}}$$
$$\left.+\frac{\{f(\tau')_{\tau'=1}-\beta'\}}{1-\{z_1-[(1/\kappa)-1]\}}\right],$$

$f(\tau)$ and $f(\tau')$ are values of β and β', respectively, for different values of τ and τ'.

$$\beta=\tan^{-1}\left\{-\frac{\partial p/\partial y_1}{\partial p/\partial x_1}\right\}_{x_1=t=-(1/\tau+1/\kappa-1)=z_1}$$

and
$$\beta'=\tan^{-1}\left\{-\frac{\partial p/\partial y_1}{\partial p/\partial x_1}\right\}_{x_1=t=(1/\tau'+1/\kappa-1)=z_1}$$

are the values of the argument of $\omega(z_1)$ on the segments $-\infty$ to $-1/\kappa$ and $1/\kappa$ to $+\infty$, respectively, and the integrals have been approximately evaluated by Simpson's rule.

The function $\omega(z_1)$ satisfies all the boundary conditions, viz. the function is real on the wall, imaginary on the segments of the unit circle, and has argument β or β' on the shock front. On the segments of the real axis $-\infty$ to $-1/\kappa$, $-1/\kappa$ to

$+1/\bar{\kappa}$ and $+1/\bar{\kappa}$ to $+\infty$ the value of $\omega(z_1)$ is obtained by substituting $\beta' = 0$, $\beta = \beta' = 0$ and $\beta = 0$, respectively.

The procedure for evaluating pressure on the wall, and numerical discussions of the problem are given in the following section.

Pressure distribution along the wall

Numerical results have been obtained for the following parameters:

p_0/p_1	α_0	α_2	$(U-q_2)/a_2$	M_2
0·5	20°	14·93°	3·02595	0·25418
0·2	20°	10·84°	3·34748	0·43670

At a point $(x, 0)$ of the wall $(-1 < x < +1)$ the co-ordinates in the Z_4-plane are given by

$$Z_4 = -\tfrac{1}{2}\pi + \tan^{-1}\left\{\frac{\{(1-x^2)(\cos^2\alpha_2 - K'^2)\}^{\frac{1}{2}}}{-Kx + \sin\alpha_2}\right\}.$$

Corresponding to points of the Z_4-plane the value of z_1 can be read with the help of the elliptic integral transformation given by (7) and by the use of Jahnke & Emde (1960). As x varies from -1 to $+1$, x_1 also varies from -1 to $+1$. The value of p is obtained at different points of the wall by the integration of the pressure derivative

$$\frac{\partial p}{\partial x_1} = \frac{\delta G[H(x_1-x_0)-1]}{x_1-x_0}\left(\frac{(x_1^2-(1/\bar{\kappa}^2))}{x_1^2-1}\right)^{\frac{1}{2}} e^{\phi},$$

where

$$\phi = \frac{x_1+[(1/\bar{\kappa})-1]}{12\pi}\left[\frac{f(\tau)_{\tau=0}}{1} + \frac{4f(\tau)_{\tau=0\cdot25}}{1+0\cdot25\{x_1+[(1/\bar{\kappa})-1]\}} + \frac{2f(\tau)_{\tau=0\cdot50}}{1+0\cdot50\{x_1+[(1/\bar{\kappa})-1]\}} + \frac{4f(\tau)_{\tau=0\cdot75}}{1+0\cdot75\{x_1+[(1/\bar{\kappa})-1]\}} + \frac{f(\tau)_{\tau=1}}{1+\{x_1+[(1/\bar{\kappa})-1]\}}\right] + \frac{x_1-[(1/\bar{\kappa})-1]}{12\pi}\left[\frac{f(\tau')_{\tau'=0}}{1} + \frac{4f(\tau')_{\tau'=0\cdot25}}{1-0\cdot25\{x_1-[(1/\bar{\kappa})-1]\}} + \frac{2f(\tau')_{\tau'=0\cdot50}}{1-0\cdot50\{x_1-[(1/\bar{\kappa})-1]\}} + \frac{4f(\tau')_{\tau'=0\cdot75}}{1-0\cdot75\{x_1-[(1/\bar{\kappa})-1]\}} + \frac{f(\tau')_{\tau'=1}}{1-\{x_1-[(1/\bar{\kappa})-1]\}}\right]$$

obtained from (14).

In figures 5 and 6 the value of

$$\frac{p_2-p_2'}{\delta(p_2-p_0)} = \frac{a_2\rho_2 q_2}{p_2-p_0}(-p/\delta)$$

has been plotted for different points of the wall. In both the cases the value of $(p_2-p_2')/[\delta(p_2-p_0)]$ increases from zero to infinity at the corner; from the corner to the point of intersection of the wall and unit circle it decreases monotonically and then maintains a constant value up to the point of intersection of the wall and reflected shock. Ter-Minassiants (1969) has brought out the effect of the angle of incidence on the pressure distribution curves for fixed shock strength.

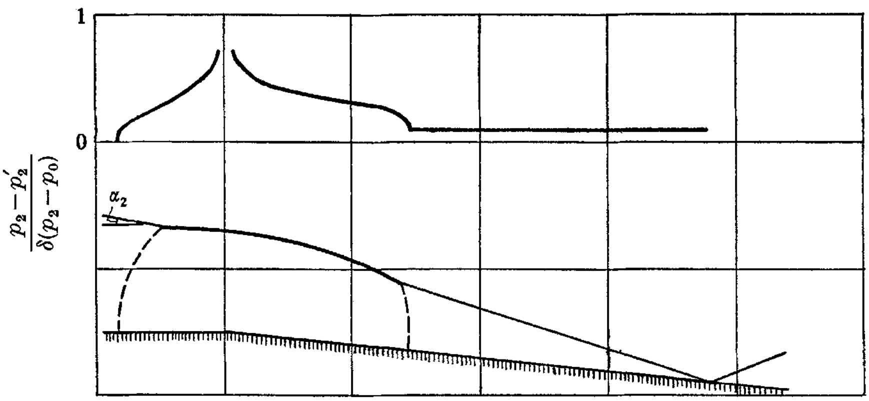

FIGURE 5. Wall pressure distribution and disturbed region ($p_1/p_0 = 2$, $\alpha_0 = 20°$, $\delta = 0{\cdot}1$ rad).

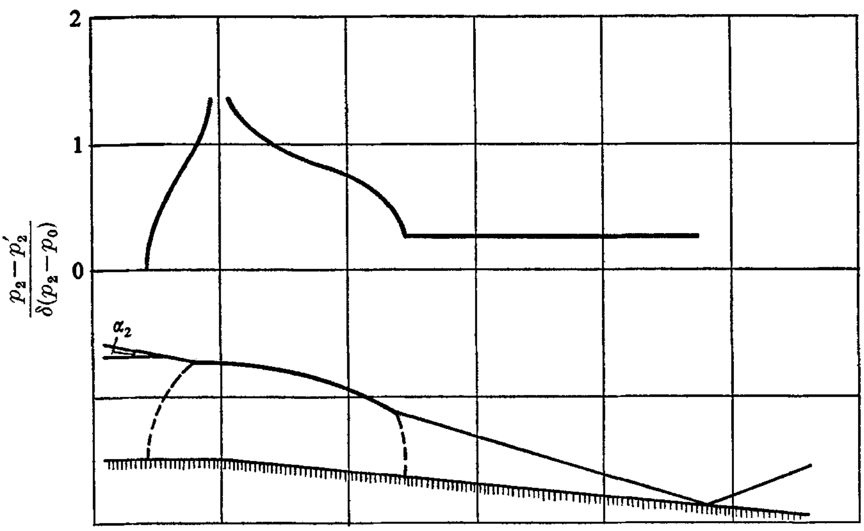

FIGURE 6. Wall pressure distribution and disturbed region ($p_1/p_0 = 5$, $\alpha_0 = 20°$, $\delta = 0{\cdot}1$ rad).

In our case, we have tried to bring out how the increase in shock strength, keeping angle of incidence fixed, results in higher pressure deficiency. A comparison between the two curves obtained shows that the value of

$$(p_2 - p_2')/\{\delta(p_2 - p_0)\}$$

grows as the shock strength is increased. Our choice of values is such that we can compare our curves with at least one of the curves of Ter-Minassiants wherein he has taken the angle of incidence to be 20°. The results of the two different approaches in regard to the oblique shock diffraction problem conform extremely well.

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